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Chapter 7
Some Types of HyperNeutrosophic Set (5): Support, Paraconsistent, Faillibilist, and Others

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Abstract

This paper builds upon the foundational advancements introduced in [14, 25–27]. The Neutrosophic Set offers a versatile mathematical framework for addressing uncertainty through its three membership functions: truth, indeterminacy, and falsity. Extensions such as the Hyperneutrosophic Set and the SuperHyperneutrosophic Set have been recently proposed to tackle increasingly sophisticated and multidimensional problems. Detailed formal definitions of these concepts can be found in [20].

In this paper, we extend various specialized classes of Neutrosophic Sets—namely, the Support Neutrosophic Set, Neutrosophic Intuitionistic Set (distinct from the Intuitionistic Fuzzy Set), Neutrosophic Paraconsistent Set, Neutrosophic Faillibilist Set, Neutrosophic Paradoxist Set, Neutrosophic Pseudo-Paradoxist Set, Neutrosophic Tautological Set, Neutrosophic Nihilist Set, Neutrosophic Dialetheist Set, and Neutrosophic Trivialist Set—by utilizing the frameworks of the Hyperneutrosophic Set and the SuperHyperneutrosophic Set.

Keywords: Set Theory, SuperhyperNeutrosophic set, Neutrosophic Set, HyperNeutrosophic set

1 Preliminaries and Definitions

This section provides an overview of the fundamental concepts and definitions essential for the discussions in this paper. The analysis utilizes classical set-theoretic operations and extends them into advanced frameworks. For readers seeking a deeper understanding of foundational set theory, resources such as [10, 36, 37, 41] are recommended. Additionally, the referenced literature offers a comprehensive exploration of the principles and applications of Neutrosophic Sets.

1.1 Neutrosophic, HyperNeutrosophic, and n-SuperHyperNeutrosophic Sets

To address uncertainty, vagueness, and imprecision in decision-making processes, numerous set-theoretic frameworks have been developed. These frameworks include Fuzzy Sets, which were introduced in foundational works such as those by Zadeh [62–70]. Another prominent framework is Intuitionistic Fuzzy Sets, extensively studied by Atanassov and others [2–7]. Vague Sets, introduced and developed by researchers, also contribute significantly to this domain [1, 8, 34, 43, 47]. Furthermore, the Hyperfuzzy Set is known as one of the extended concepts of the Fuzzy Set [9, 13, 22, 23, 35, 38–40, 42, 44, 60].

Neutrosophic Sets, first introduced by Smarandache, offer a powerful means of capturing indeterminacy, allowing for more nuanced decision-making models [16–18, 24, 28–33, 51, 52, 58]. Neutrosophic Sets generalize Fuzzy Sets by introducing an additional component: indeterminacy, alongside truth and falsity [49–52]. This enhancement allows for a richer and more precise representation of uncertainty and ambiguity.

To address increasingly complex scenarios, HyperNeutrosophic Sets and n-SuperHyperNeutrosophic Sets have been developed. These advanced models are particularly suited for high-dimensional and intricate problem spaces [15, 20, 55]. Relevant definitions and simple examples are provided below.

Definition 1.1 (Base Set)**.** A *base set* S is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of S.

Definition 1.2 (Powerset)**.** [19, 46] The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 1.3 (n-th Powerset)**.** (cf. [11, 19, 21, 48, 56])

The n-th powerset of a set H, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the powerset of H with the empty set removed.

Definition 1.4 (Neutrosophic Set)**.** [51, 52] Let X be a non-empty set. A *Neutrosophic Set (NS) A* on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Example 1.5 (Neutrosophic Set)**.** *Scenario:* Assessing public opinion on a controversial policy.

Example: Let $X = \{\text{Alice, Bob, Charlie}\}$, representing individuals with varying opinions on the policy. The membership functions represent their support (T), uncertainty (I), and opposition (F) as follows:

- For Alice: $T_A(\text{Alice}) = 0.8$ (80% support), $I_A(\text{Alice}) = 0.1$ (10% uncertain), $F_A(\text{Alice}) = 0.1$ (10% oppose).
- For Bob: $T_A(\text{Bob}) = 0.5$, $I_A(\text{Bob}) = 0.3$, $F_A(\text{Bob}) = 0.2$.
- For Charlie: $T_A(\text{Charlie}) = 0.3$, $I_A(\text{Charlie}) = 0.4$, $F_A(\text{Charlie}) = 0.3$.

This representation allows nuanced analysis, reflecting both certainty and uncertainty in opinions.

Definition 1.6 (HyperNeutrosophic Set)**.** (cf. [12, 15, 20, 22, 55]) Let X be a non-empty set. A *HyperNeutrosophic Set (HNS) $\tilde{A}$* on X is a mapping:

$$\tilde{\mu} : X \to \mathcal{P}([0,1]^3),$$

where $\mathcal{P}([0,1]^3)$ is the family of all non-empty subsets of the unit cube $[0,1]^3$. For each $x \in X$, $\tilde{\mu}(x) \subseteq [0,1]^3$ is a set of neutrosophic membership triplets (T, I, F) that satisfy:

$$0 \leq T + I + F \leq 3.$$

Example 1.7 (HyperNeutrosophic Set)**.** *Scenario:* Analyzing customer satisfaction for multiple products, considering evaluations from different dimensions or individuals.

Example: Let $X = \{\text{Product A, Product B}\}$, where each product has multi-dimensional satisfaction scores represented by sets of neutrosophic triplets:

- For Product A:
$$\tilde{\mu}(\text{Product A}) = \{(0.8, 0.1, 0.1), (0.7, 0.2, 0.1)\},$$
representing two customers' evaluations where each triplet denotes degrees of truth, indeterminacy, and falsity.

- For Product B:
$$\tilde{\mu}(\text{Product B}) = \{(0.6, 0.3, 0.1), (0.5, 0.4, 0.1)\}.$$

This structure enables richer analysis by aggregating diverse customer feedback for a comprehensive view.

Definition 1.8 (n-SuperHyperNeutrosophic Set)**.** (cf. [12, 15, 20, 22]) Let X be a non-empty set. An *n-SuperHyperNeutrosophic Set (n-SHNS)* is a recursive generalization of Neutrosophic Sets and HyperNeutrosophic Sets. It is defined as a mapping:

$$\tilde{A}_n : \mathcal{P}_n(X) \to \mathcal{P}_n([0,1]^3),$$

where:

- $\mathcal{P}_1(X) = \mathcal{P}(X)$, the power set of X, and for $k \geq 2$,
$$\mathcal{P}_k(X) = \mathcal{P}(\mathcal{P}_{k-1}(X)),$$
representing the k-th nested family of non-empty subsets of X.
- $\mathcal{P}_n([0,1]^3)$ is defined similarly for the unit cube $[0,1]^3$.

For each $A \in \mathcal{P}_n(X)$ and $(T, I, F) \in \tilde{A}_n(A)$, the following condition is satisfied:

$$0 \leq T + I + F \leq 3,$$

where T, I, F represent the degrees of truth, indeterminacy, and falsity for the n-th level subsets of X.

Example 1.9 (n-SuperHyperNeutrosophic Set)**.** *Scenario:* Multi-level hierarchical analysis of climate change impacts.

Example: Let $X = \{\text{Temperature, Rainfall, Sea Level}\}$, representing key factors influenced by climate change. We consider a three-level hierarchy:

- *Level 1:* Regions {Region 1, Region 2}.
- *Level 2:* Countries within regions, e.g., {Country A, Country B, Country C}.
- *Level 3:* Cities within countries, e.g., {City X, City Y, City Z}.

For each level, the n-SuperHyperNeutrosophic Set assigns a family of subsets with membership triplets. For instance:
$$\tilde{A}_3(\text{City X}) = \{(0.8, 0.15, 0.05), (0.7, 0.2, 0.1)\},$$
where each triplet represents truth (T), indeterminacy (I), and falsity (F) degrees at the city level. This approach integrates uncertainty at regional, country, and city scales for holistic decision-making.

2 Results of This Paper

This section outlines the main results presented in this paper.

2.1 Support-Neutrosophic set

A Support-Neutrosophic Set extends neutrosophic sets by adding a support membership function, modeling truth, indeterminacy, falsity, and support degrees [45, 61].

Definition 2.1 (Support-Neutrosophic set)**.** [61] Let U be a universal set. A *Support-Neutrosophic Set* (SNS) A on U is characterized by four membership functions:

$$A = \{(x, T_A(x), I_A(x), F_A(x), s_A(x)) \mid x \in U\},$$

where:

- $T_A(x)$ is the *truth-membership function*,
- $I_A(x)$ is the *indeterminacy-membership function*,
- $F_A(x)$ is the *falsity-membership function*,
- $s_A(x)$ is the *support-membership function*.

Each membership function satisfies:

$$T_A(x), I_A(x), F_A(x), s_A(x) \in [0, 1] \quad \text{for all } x \in U.$$

There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$, so:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3,$$

and:

$$0 \leq s_A(x) \leq 1.$$

Definition 2.2 (Support HyperNeutrosophic Set (SHNS))**.** Let X be a non-empty set. A *Support HyperNeutrosophic Set* $\widehat{A}$ on X is defined as a mapping

$$\widehat{\mu}\colon X \longrightarrow \mathcal{P}\big([0, 1]^4\big),$$

where $\mathcal{P}([0, 1]^4)$ is the family of all non-empty subsets of the 4-dimensional unit hypercube $[0, 1]^4$. For each $x \in X$, $\widehat{\mu}(x) \subseteq [0, 1]^4$ is a set of quadruples $\big(T, I, F, s\big)$, where

$$(T, I, F, s) \in [0, 1]^4,$$

subject to the following neutrosophic-like constraint on (T, I, F):

$$0 \leq T + I + F \leq 3,$$

and the additional *support* coordinate $s \in [0, 1]$ is unrestricted apart from lying in $[0, 1]$.

Hence, each $x \in X$ can have multiple possible quadruples $\big(T, I, F, s\big)$, each representing degrees of *truth*, *indeterminacy*, *falsity*, and *support*, respectively, in a hyper-collection manner.

Theorem 2.3. *Let $\widehat{A}$ be a Support HyperNeutrosophic Set. Then:*

1. *If for every $x \in X$, $\widehat{\mu}(x)$ is a singleton, i.e. $\widehat{\mu}(x) = \{\big(T, I, F, s\big)\}$, we recover a* Support Neutrosophic Set*.*
2. *If we exclude the support coordinate (or fix it as a constant), we recover a* HyperNeutrosophic Set*.*

Hence, the concept of a Support HyperNeutrosophic Set generalizes both the Support Neutrosophic Set and the HyperNeutrosophic Set.

Proof. *(1) Reduction to Support Neutrosophic Set:* When $\widehat{\mu}(x)$ is restricted to exactly one quadruple (T, I, F, s) per $x \in X$, we have a single 4-tuple for each x. This precisely matches the usual definition of an SNS (where each x has membership degrees $T_A(x)$, $I_A(x)$, $F_A(x)$, and $s_A(x) \in [0, 1]$ with $T_A(x) + I_A(x) + F_A(x) \leq 3$).

(2) Reduction to HyperNeutrosophic Set: If we fix $s = 0$ or $s = 1$ (or remove s altogether), then $\widehat{\mu}(x) \subseteq [0, 1]^3$ for each x, and we keep the condition $T + I + F \leq 3$. This is exactly the definition of a HyperNeutrosophic Set $\tilde{A}$.

Therefore, $\widehat{A}$ unifies both structures in a single framework, completing the proof. □

Definition 2.4 (Support n-SuperHyperNeutrosophic Set (n-SHNS with Support))**.** Let X be a non-empty set. An *Support* n-SuperHyperNeutrosophic Set (abbreviated $\widehat{A}_n$) is defined as a mapping

$$\widehat{A}_n : \mathcal{P}_n(X) \longrightarrow \mathcal{P}_n\big([0, 1]^4\big),$$

where:

- $\mathcal{P}_1(X) = \mathcal{P}(X)$, and for $k \geq 2$,
$$\mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big),$$
representing nested families of non-empty subsets of X up to depth k.
- $\mathcal{P}_n([0, 1]^4)$ is defined similarly for subsets of the 4-dimensional unit hypercube $[0, 1]^4$.

For each $A \in \mathcal{P}_n(X)$ and each quadruple $(T, I, F, s) \in \widehat{A}_n(A)$, we require:

$$T, I, F, s \in [0, 1], \quad 0 \leq T + I + F \leq 3.$$

That is, the first three coordinates (T, I, F) represent truth, indeterminacy, and falsity degrees (with a neutrosophic constraint), and the fourth coordinate $s \in [0, 1]$ represents a *support* degree. The "n-superhyper" aspect means we interpret $\widehat{A}_n$ at successively deeper levels of subsets in $\mathcal{P}_n(X)$.

Theorem 2.5. *(Unification of Support HyperNeutrosophic Set and n-SuperHyperNeutrosophic Set)*

Let $\widehat{A}_n$ be a Support n-SuperHyperNeutrosophic Set as in Definition 2.4. Then:

1. *If $n = 1$, $\widehat{A}_1$ reduces to a* Support HyperNeutrosophic Set *(see Definition 2.2), where each element in $\mathcal{P}_1(X) = \mathcal{P}(X)$ is just a subset $A \subseteq X$, and $\widehat{A}_1(A) \subseteq [0, 1]^4$.*
2. *If we remove the extra support coordinate from Definition 2.4, we recover the standard n-*SuperHyperNeutrosophic Set.

Thus, a Support n-SuperHyperNeutrosophic Set generalizes both the Support HyperNeutrosophic Set (when $n = 1$) and the n-SuperHyperNeutrosophic Set (when we remove or fix the support dimension).

Proof. *(1) Case $n = 1$:* By Definition 2.4, for $n = 1$ we map each $A \in \mathcal{P}_1(X) = \mathcal{P}(X)$ to a subset $\widehat{A}_1(A) \subseteq [0, 1]^4$. But each element of $\mathcal{P}(X)$ is just a subset of X. In practice, we can equate each $A \subseteq X$ with an element $x \in X$ if we want an element-wise perspective, obtaining precisely a "hyper-collection" of quadruples (T, I, F, s) for each x. That matches the structure of a Support HyperNeutrosophic Set.

(2) Removing the support dimension: If we ignore the fourth coordinate s, then each quadruple (T, I, F, s) reduces to $(T, I, F) \in [0, 1]^3$. The condition $T+I+F \leq 3$ yields exactly the standard n-SuperHyperNeutrosophic constraint. This proves that the new notion unifies both concepts in a single framework. □

2.2 Special Cases of Neutrosophic Sets

This subsection provides an explanation of the Special Cases of Neutrosophic Sets. The Neutrosophic Set can be transformed into various specialized set concepts, such as the Neutrosophic Intuitionistic Set (distinct from the Intuitionistic Fuzzy Set), Neutrosophic Paraconsistent Set, Neutrosophic Faillibilist Set, Neutrosophic Paradoxist Set, Neutrosophic Pseudo-Paradoxist Set, Neutrosophic Tautological Set, Neutrosophic Nihilist Set, Neutrosophic Dialetheist Set, and Neutrosophic Trivialist Set (cf. [53, 54, 57, 59]).

These concepts can be generalized using the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set frameworks. Due to the extensive nature of the proofs, they are omitted in this paper. For further details, refer to relevant sources such as [20].

Definition 2.6 (Non-Standard Unit Interval)**.** Let

$$]-0, 1+[\; = \; \{x \mid -0 \leq x \leq 1+\}$$

be the so-called *non-standard unit interval*, which can include "infinitesimal" parts below 0 (denoted by -0) and possibly "infinite" or "beyond 1" parts above 1 (denoted by $1+$). In various treatments, one may restrict to the standard interval $[0, 1]$. However, in the most general neutrosophic sense, membership degrees can lie in this broader range $]-0, 1+[$.

Remark 2.7. Throughout, for each element x in the universe U (or X, S, etc.), we associate three subsets (or sub-values) $T, I, F \subseteq]-0, 1+[$. Intuitively:

$$T = \text{(truth-degree subset)}, \quad I = \text{(indeterminacy-degree subset)}, \quad F = \text{(falsity-degree subset)}.$$

We often denote an element x by $x(T, I, F)$, signifying that x has partial membership characterized by (T, I, F).

The Neutrosophic Set is redefined using the non-standard unit interval. The definition is provided below.

Definition 2.8 (Neutrosophic Set (Using Non-standard unit interval))**.** [51, 52, 58] Let U be a universe of discourse, and let $M \subseteq U$. A *Neutrosophic Set* M is defined by assigning to each $x \in U$ an ordered triple (T_x, I_x, F_x), where

$$T_x, I_x, F_x \subseteq]-0, 1+[\quad \text{(the non-standard unit interval)},$$

representing the *truth*, *indeterminacy*, and *falsity* percentages (or degrees) of x belonging to M. Concretely, we write:

$$x(T_x, I_x, F_x).$$

These three subsets must satisfy the broad neutrosophic condition that

$$-0 \leq \inf(T_x) + \inf(I_x) + \inf(F_x) \quad \text{and} \quad \sup(T_x) + \sup(I_x) + \sup(F_x) \leq 3+,$$

allowing the sum of (truth + indeterminacy + falsity) to be anywhere in the extended range up to 3+, and similarly down to -0.

In the special (standard) case where each of T_x, I_x, F_x is a singleton in $[0, 1]$, one recovers simpler forms such as fuzzy, intuitionistic fuzzy, or other sets. But the full neutrosophic set framework permits negative infinitesimals or values beyond 1, depending on the chosen non-standard extension.

This definition generalizes the classical set ($T = 1$, $I = 0$, $F = 0$), fuzzy set ($T \in [0, 1]$, $I = 0$, $F = 1 - T$), intuitionistic fuzzy set, paraconsistent set, and many others (see discussions below).

Definition 2.9 (Hyperneutrosophic General Form)**.** Let U be the universe of discourse. A *Hyperneutrosophic Set* $\mathcal{H}$ assigns to each element $x \in U$ a triple

$$\big(\mathcal{T}(x), \; \mathcal{I}(x), \; \mathcal{F}(x)\big),$$

where each of $\mathcal{T}(x)$, $\mathcal{I}(x)$, $\mathcal{F}(x)$ is a *hyper-collection* of membership degrees (or subsets of membership degrees) in the non-standard interval $]-0, 1+[$. Concretely, we might view

$$\mathcal{T}(x) \subseteq \mathcal{P}\big(]-0, 1+[\big), \quad \mathcal{I}(x) \subseteq \mathcal{P}\big(]-0, 1+[\big), \quad \mathcal{F}(x) \subseteq \mathcal{P}\big(]-0, 1+[\big).$$

(Or alternatively each could be a function from some index set Λ to $]-0, 1+[$.)

We then impose the neutrosophic constraints in a *hyper* sense: for all choices of $\tau \in \mathcal{T}(x), \iota \in \mathcal{I}(x), \varphi \in \mathcal{F}(x)$, one must satisfy the usual bounds

$$-0 \leq \inf(\tau) + \inf(\iota) + \inf(\varphi), \quad \sup(\tau) + \sup(\iota) + \sup(\varphi) \leq 3+,$$

plus whatever extra condition the special class imposes.

Notation: We denote such a set as $\mathcal{H}: x \mapsto (\mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x))$.

Remark 2.10. All the special classes (Intuitionistic, Paraconsistent, etc.) can be "hyperized" by requiring that each $\mathcal{T}(x)$, $\mathcal{I}(x)$, $\mathcal{F}(x)$ satisfies the relevant constraints. For example, a *HyperNeutrosophic-Intuitionistic Set* might demand $\sup(\tau) + \sup(\iota) + \sup(\varphi) < 1$ for all $\tau \in \mathcal{T}(x)$, $\iota \in \mathcal{I}(x)$, etc.

Definition 2.11 (n-SuperHyperneutrosophic General Form)**.** An *n-SuperHyperneutrosophic Set* $\mathcal{H}^{(n)}$ extends Definition 2.9 to *n-level* hyper-memberships. At level 1, we assign $(\mathcal{T}_1(x), \mathcal{I}_1(x), \mathcal{F}_1(x))$. For each element $\tau \in \mathcal{T}_1(x)$, we define a second-level triple $(\mathcal{T}_2(\tau), \mathcal{I}_2(\tau), \mathcal{F}_2(\tau))$, etc., up to level n. Each level enforces the standard or special neutrosophic constraints in a hyper sense. Symbolically,

$$\mathcal{H}^{(n)}(x) = (\mathcal{T}_1(x),\ \mathcal{I}_1(x),\ \mathcal{F}_1(x)),$$

$$\text{and for each } \tau \in \mathcal{T}_1(x), \quad (\mathcal{T}_2(\tau),\ \mathcal{I}_2(\tau),\ \mathcal{F}_2(\tau)),$$

$$\vdots$$

$$\text{Level } n \text{ similarly.}$$

All the specialized conditions (e.g., $\sup(T) + \sup(I) + \sup(F) < 1$, or $\inf(I) > 0$, etc.) must hold across *all relevant sublevels* to preserve the special-case property in the n-SuperHyper sense.

Remark 2.12. Just like Hyperneutrosophic Sets, any of the specialized classes (Intuitionistic, Paraconsistent, etc.) can be "n-superhyperized." For instance, an *n-SuperHyperNeutrosophic Intuitionistic Set* would impose sup of each triple's sum < 1 across all n-levels of membership data, and so on.

2.2.1 Neutrosophic Intuitionistic Set

A Neutrosophic Intuitionistic Set generalizes Intuitionistic Fuzzy Sets by requiring that the supremum values of truth, indeterminacy, and falsity strictly sum to less than 1. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.13 (Neutrosophic Intuitionistic Set)**.** [53] An *Neutrosophic Intuitionistic Set* is a special class of neutrosophic set in which each element $x(T, I, F)$ satisfies

$$\sup(T) + \sup(I) + \sup(F) < 1.$$

Hence the total "sum" of truth, indeterminacy, and falsity is strictly below 1 when we consider the supremum values. This models *incomplete* knowledge about membership: no matter how large you make truth, indeterminacy, or falsity, they cannot jointly reach 1.

By comparison, classical *intuitionistic fuzzy sets* (in the sense of Atanassov) typically require $T + F \leq 1$, with an *indeterminacy* of $1 - (T + F)$. The above neutrosophic condition generalizes that notion using possibly non-standard intervals.

Definition 2.14 (HyperNeutrosophic-Intuitionistic Set)**.** A *HyperNeutrosophic Intuitionistic Set* is a hyperneutrosophic set $\mathcal{H}$ such that for every $x \in U$ and for all $\tau \in \mathcal{T}(x)$, $\iota \in \mathcal{I}(x)$, $\varphi \in \mathcal{F}(x)$, the sum of supremum values satisfies

$$\sup(\tau) + \sup(\iota) + \sup(\varphi) < 1.$$

We can similarly impose that the sum of $\inf(\tau) + \inf(\iota) + \inf(\varphi)$ remains below 1, depending on the exact formalism. This ensures that each sub-element in the hyper-collection respects the "incomplete membership" principle of the original intuitionistic concept, but now in a hyper sense.

Definition 2.15 (n-SuperHyperNeutrosophic Intuitionistic Set)**.** An *n-SuperHyperNeutrosophic Intuitionistic Set* extends Definition by enforcing the same $\sup(\tau) + \sup(\iota) + \sup(\varphi) < 1$ (or similar) constraint at *each* of the n levels of membership. In other words, at level 1, for each $\tau_1 \in \mathcal{T}_1(x)$, $\iota_1 \in \mathcal{I}_1(x)$, $\varphi_1 \in \mathcal{F}_1(x)$, we require

$$\sup(\tau_1) + \sup(\iota_1) + \sup(\varphi_1) < 1,$$

and for level 2, each $\tau_2 \in \mathcal{T}_2(\tau_1)$, $\iota_2 \in \mathcal{I}_2(\iota_1)$, etc., must also satisfy the same incomplete sum constraint, and so on up to level n. This hierarchical layering captures multi-stage or multi-dimensional incomplete knowledge.

2.2.2 Neutrosophic Paraconsistent Set

A Neutrosophic Paraconsistent Set captures overlapping information by requiring truth, indeterminacy, and falsity supremum values to exceed 1. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.16 (Neutrosophic Paraconsistent Set)**.** [53] A *Neutrosophic Paraconsistent Set* is a special class of neutrosophic set in which each element $x(T, I, F)$ satisfies

$$\sup(T) + \sup(I) + \sup(F) \ > \ 1.$$

In other words, the total supremum of (T, I, F) strictly exceeds 1, capturing *paraconsistent information*, where one can have overlapping degrees that go beyond what is normally considered a single unity. This is closely tied to paraconsistent logic, where contradictions can coexist without trivialization.

Definition 2.17 (HyperNeutrosophic Paraconsistent Set)**.** A *HyperNeutrosophic Paraconsistent Set* is a hyperneutrosophic set $\mathcal{H}$ where each sub-element triple (τ, ι, φ) satisfies

$$\sup(\tau) + \sup(\iota) + \sup(\varphi) \ > \ 1.$$

Equivalently, the total membership across truth, indeterminacy, and falsity exceeds 1 in each hyper-subset. This extends the classical paraconsistent property ($\sup(T) + \sup(I) + \sup(F) > 1$) to every layer in the hyper-collection.

Definition 2.18 (n-SuperHyperNeutrosophic Paraconsistent Set)**.** An *n-SuperHyperNeutrosophic Paraconsistent Set* enforces the paraconsistent condition at *each of the n membership levels*. That is, for any element $x \in U$, at level k (where $1 \le k \le n$), each triple $(\tau_k, \iota_k, \varphi_k)$ satisfies

$$\sup(\tau_k) + \sup(\iota_k) + \sup(\varphi_k) > 1.$$

This captures a multi-layer logic scenario where paraconsistent overlap is present through all nested or recursive membership steps.

2.2.3 Neutrosophic Faillibilist Set

A Neutrosophic Faillibilist Set ensures every element has a strictly positive lower bound of indeterminacy, capturing inherent uncertainty. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.19 (Neutrosophic Faillibilist Set)**.** [53] A *Neutrosophic Faillibilist Set* is a class of neutrosophic set in which every element $x(T, I, F)$ has

$$\inf(I) \ > \ 0.$$

This means each element has a strictly positive lower bound of *indeterminacy*. In effect, no element is fully known (there is always some irreducible uncertainty).

Definition 2.20 (HyperNeutrosophic Faillibilist Set)**.** A *HyperNeutrosophic Faillibilist Set* is a hyperneutrosophic set $\mathcal{H}$ such that, for every $x \in U$, each $\iota \in \mathcal{I}(x)$ satisfies

$$\inf(\iota) > 0.$$

Hence, all sub-members for indeterminacy remain strictly above 0, generalizing the classic $\inf(I) > 0$ requirement to the hyper context.

Definition 2.21 (n-SuperHyperNeutrosophic Faillibilist Set)**.** An *n-SuperHyperNeutrosophic Faillibilist Set* extends the above property to n levels: at level k, each ι_k (for $\iota_k \in \mathcal{I}_k$) must have $\inf(\iota_k) > 0$. This ensures a permanent positive minimal indeterminacy across all nested membership layers.

2.2.4 Neutrosophic Paradoxist Set

A *Neutrosophic Paradoxist Set* is a class of neutrosophic set in which every element $x(T, I, F)$ has the specific form $x(1,\ I,\ 1)$. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.22 (Neutrosophic Paradoxist Set)**.** [53] A *Neutrosophic Paradoxist Set* is a class of neutrosophic set in which every element $x(T, I, F)$ has the specific form

$$x(1,\ I,\ 1).$$

Interpreted literally, each element belongs 100% to the set *and* does not belong 100% at the same time. Formally, $\inf(T) \geq 1$ (or T includes 1) and $\inf(F) \geq 1$. The indeterminacy I can be anything, but typically $I \subseteq]-0, 1+[$ as usual. This embodies a *paradox*: total membership and total non-membership simultaneously.

Definition 2.23 (HyperNeutrosophic Paradoxist Set)**.** A *HyperNeutrosophic Paradoxist Set* is a hyperneutrosophic set $\mathcal{H}$ where every sub-element triple (τ, ι, φ) satisfies

$$\tau \text{ contains } 1, \quad \varphi \text{ contains } 1.$$

In other words, $\inf(\tau) \geq 1$ and $\inf(\varphi) \geq 1$. This enforces "complete membership" and "complete non-membership" simultaneously at the hyper level. The indeterminacy ι can vary.

Definition 2.24 (n-SuperHyperNeutrosophic Paradoxist Set)**.** An *n-SuperHyperNeutrosophic Paradoxist Set* applies the condition $\inf(\tau_k) \geq 1$ and $\inf(\varphi_k) \geq 1$ at each level $k = 1, \ldots, n$. Thus, from the first to the nth membership layer, every sub-triple is paradoxical (full membership and full non-membership).

2.2.5 Neutrosophic Pseudo-Paradoxist Set

A Neutrosophic Pseudo-Paradoxist Set is a neutrosophic set where elements exhibit "partially total" membership or non-membership: one dimension is fully determined (100%), while the other is partially defined. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.25 (Neutrosophic Pseudo-Paradoxist Set)**.** [53] A *Neutrosophic Pseudo-Paradoxist Set* is a class of neutrosophic set in which every element $x(T, I, F)$ satisfies one of the following forms:

$$x(1,\ I,\ F) \quad \text{with} \quad 0 < \inf(F) \leq \sup(F) < 1,$$

$$\text{or} \quad x(T,\ I,\ 1) \quad \text{with} \quad 0 < \inf(T) \leq \sup(T) < 1.$$

Hence we have "partially total" membership or non-membership combined with partial membership. Concretely:

- In the first form, the element belongs 100% ($\inf(T) \geq 1$) and also partially does not belong (some fraction $F \in (0, 1)$).
- In the second form, the element partially belongs $T \in (0, 1)$ but also does not belong 100%.

This generalizes the idea of a paradox, but not at the "full 1 and full 1" for membership and non-membership. Instead, membership is fully 1 in one dimension, while the other dimension is strictly between 0 and 1, or vice versa.

Definition 2.26 (HyperNeutrosophic Pseudo-Paradoxist Set)**.** A *HyperNeutrosophic Pseudo-Paradoxist Set* is a hyperneutrosophic set such that, for every $\tau \in \mathcal{T}(x)$, $\varphi \in \mathcal{F}(x)$, either

$$\inf(\tau) \geq 1 \quad \text{and} \quad 0 < \inf(\varphi) \leq \sup(\varphi) < 1$$

or

$$0 < \inf(\tau) \leq \sup(\tau) < 1 \quad \text{and} \quad \inf(\varphi) \geq 1$$

for each hyper-subset. This extends the "pseudo-paradoxical" partial membership and partial non-membership to all sub-levels in the hyper-collection.

Definition 2.27 (n-SuperHyperNeutrosophic Pseudo-Paradoxist Set)**.** An *n-SuperHyperNeutrosophic Pseudo-Paradoxist Set* repeats these partial conditions $\big(1,\ F \in (0, 1)\big)$ or $\big(T \in (0, 1),\ 1\big)$ at each membership level k. Concretely, if $\tau_k \in \mathcal{T}_k, \varphi_k \in \mathcal{F}_k$, then either $\inf(\tau_k) \geq 1$ and $\sup(\varphi_k) < 1$, or vice versa, for each level k.

2.2.6 Neutrosophic Tautological Set

A Neutrosophic Tautological Set is a neutrosophic set where every element absolutely belongs to the set (T¿1) with no indeterminacy or falsity. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.28 (Neutrosophic Tautological Set)**.** [53] A *Neutrosophic Tautological Set* is a class of neutrosophic set in which every element x has the form

$$x(1+, -0, -0),$$

meaning it *absolutely* belongs to the set in all possible worlds/scenarios. Symbolically, $\inf(T) \geq 1+$ (an extended value beyond 1), while $\sup(I) \leq -0$ and $\sup(F) \leq -0$, i.e. no indeterminacy and no falsity even in extended sense. This is a "universal truth" membership scenario, hence "tautological."

Definition 2.29 (HyperNeutrosophic Tautological Set)**.** A *HyperNeutrosophic Tautological Set* is one in which, for every $\tau \in \mathcal{T}(x)$, we have $\inf(\tau) \geq 1+$, and simultaneously $\inf(\iota) \leq -0$ and $\inf(\varphi) \leq -0$ for all ι, φ in $\mathcal{I}(x)$, $\mathcal{F}(x)$. Thus each sub-collection ensures absolute truth across the entire hyper-structure.

Definition 2.30 (n-SuperHyperNeutrosophic Tautological Set)**.** An *n-SuperHyperNeutrosophic Tautological Set* demands that, at each level k, the truth sub-collection has elements with $\inf(\tau_k) \geq 1+$ and the indeterminacy/falsity sub-collections remain ≤ -0. Thus, from level 1 to level n, we have absolute membership, with no possibility of partial or negative membership.

2.2.7 Neutrosophic Nihilist Set

A Neutrosophic Nihilist Set is a neutrosophic set where every element absolutely does not belong (F¿1) with no truth or indeterminacy. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.31 (Neutrosophic Nihilist Set)**.** [53] A *Neutrosophic Nihilist Set* is a class of neutrosophic set in which every element x has the form

$$x(-0, -0, 1+),$$

meaning it *absolutely does not* belong to the set in all possible worlds. Symbolically, $\inf(F) \geq 1+$ (falsity beyond 1), while $\sup(T) \leq -0$ and $\sup(I) \leq -0$, i.e. no truth and no indeterminacy. The empty set is a particular case of a nihilist set.

Definition 2.32 (HyperNeutrosophic Nihilist Set)**.** A *HyperNeutrosophic Nihilist Set* ensures $\inf(\varphi) \geq 1+$ for each $\varphi \in \mathcal{F}(x)$, while $\sup(\tau) \leq -0$ and $\sup(\iota) \leq -0$. In every hyper-subset, the element absolutely does not belong, across all sub-members.

Definition 2.33 (n-SuperHyperNeutrosophic Nihilist Set)**.** An *n-SuperHyperNeutrosophic Nihilist Set* repeats the $\inf(\varphi_k) \geq 1+$ condition at each of the n membership levels, ensuring total falsity and no truth/indeterminacy for all nested sub-layers.

2.2.8 Neutrosophic Dialetheist Set

A Neutrosophic Dialetheist Set allows elements to belong simultaneously to the set and its complement, modeling logical contradictions. This concept can be extended using the frameworks of the Hyperneutrosophic Set and n-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.34 (Neutrosophic Dialetheist Set)**.** [53] A *Neutrosophic Dialetheist Set* is a class of neutrosophic set that models a situation where some element(s) also belong to the complement of the set. Formally, there exists at least one element $x(T, I, F)$ in the set M such that x also belongs to the complement $C(M)$. Equivalently, there is an overlap between M and its complement for at least one x. In neutrosophic terms, one might express this by saying T for membership in M is non-zero (or high), and simultaneously T for membership in $C(M)$ is also non-zero. This is akin to dialetheism in logic, where contradictions can be true.

Definition 2.35 (HyperNeutrosophic Dialetheist Set)**.** A *HyperNeutrosophic Dialetheist Set* is a hyperneutrosophic set $\mathcal{H}$ where there exists at least one element $x \in U$ (and at least one sub-membership triple $\tau \in \mathcal{T}(x)$, etc.) that also belongs to the complement's hyper-sub-collection. In practice, this means at least one level of membership for x has $\tau > 0$ for both the set and its complement, reflecting the dialetheist notion that contradictory membership can be valid.

Definition 2.36 (n-SuperHyperNeutrosophic Dialetheist Set)**.** We say $\mathcal{H}^{(n)}$ is an *n-SuperHyperNeutrosophic Dialetheist Set* if, at some level(s) k, there is an element that belongs simultaneously to both $\mathcal{H}^{(n)}$ and its n-level complement. In other words, the contradiction is "allowed" or "realized" across (possibly multiple) sub-layers of membership data.

2.2.9 Neutrosophic Trivialist Set

A Neutrosophic Trivialist Set includes all its elements in both the set and its complement, representing universal contradictions. This concept can be extended using the frameworks of the Hyperneutrosophic Set and *n*-SuperHyperneutrosophic Set. The formal definition is provided below.

Definition 2.37 (Neutrosophic Trivialist Set)**.** [53] A *Neutrosophic Trivialist Set* is a class of neutrosophic set where every element also belongs to the complement. Formally, for every $x(T, I, F)$ in M, x is also in $C(M)$. That is, the intersection between M and $C(M)$ is not only non-empty, but actually contains all elements of M. In a classical sense, "everything is true" and "everything is false" at once. Trivialism is the position that all contradictions are in fact the case.

Definition 2.38 (HyperNeutrosophic Trivialist Set)**.** A *HyperNeutrosophic Trivialist Set* ensures that *every* element in M also belongs to the complement's hyper-collection. Formally, for every $x \in M$ and every triple $\tau \in \mathcal{T}(x)$, etc., there is a triple in the complement's hyper-collection that also certifies membership. This generalizes the trivialist idea that $\cap(M, C(M))$ is universal across all sub-members.

Definition 2.39 (n-SuperHyperNeutrosophic Trivialist Set)**.** An *n-SuperHyperNeutrosophic Trivialist Set* extends this universal overlap to all membership levels: from level 1 to level n, each sub-triple for any $x \in M$ is repeated in the membership structure of the complement, thus making everything "trivially" shared across all levels.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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